VLDB 2025 – Research track: Data Stream Mining

Approximate 2-hop neighborhoods on incremental graphs: An efficient lazy approach

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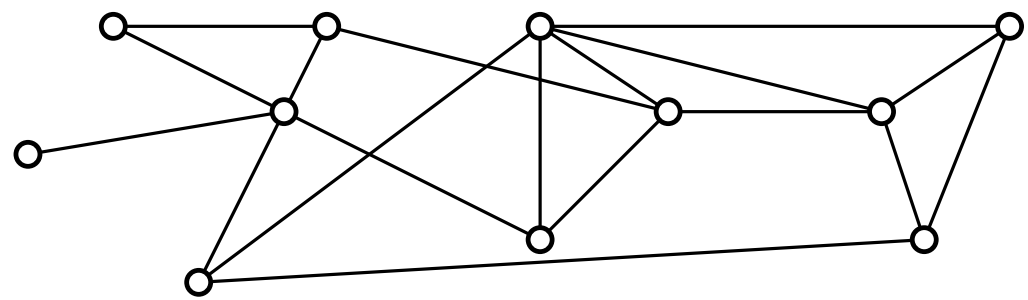
General Framework

Layer 0

Underlying *dynamic* network



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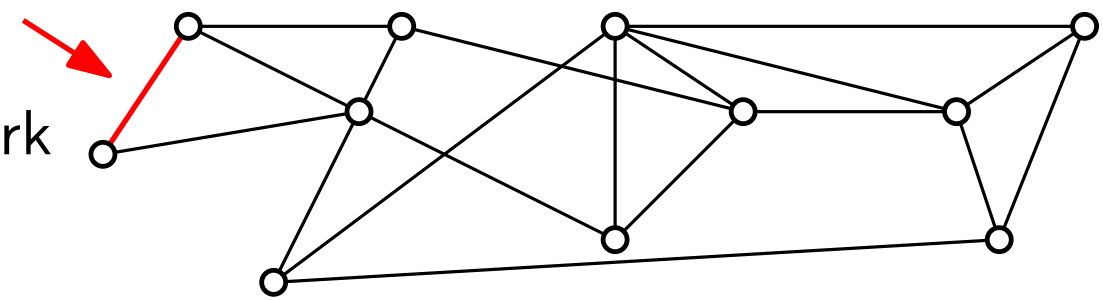

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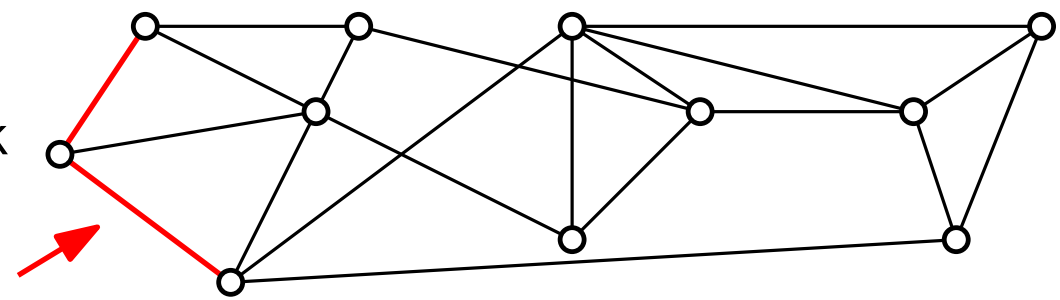

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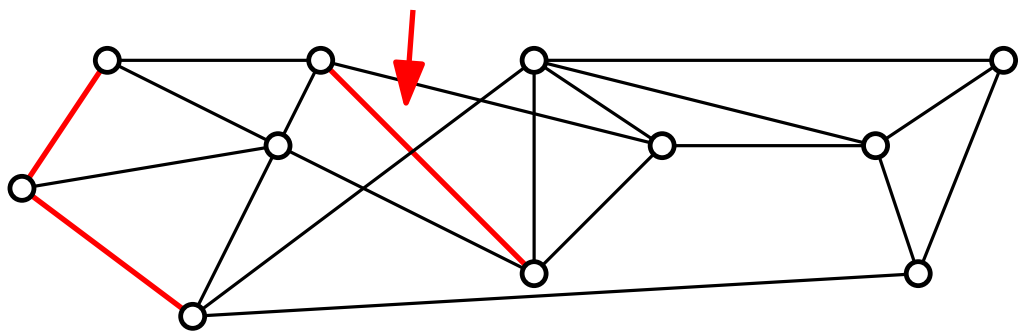

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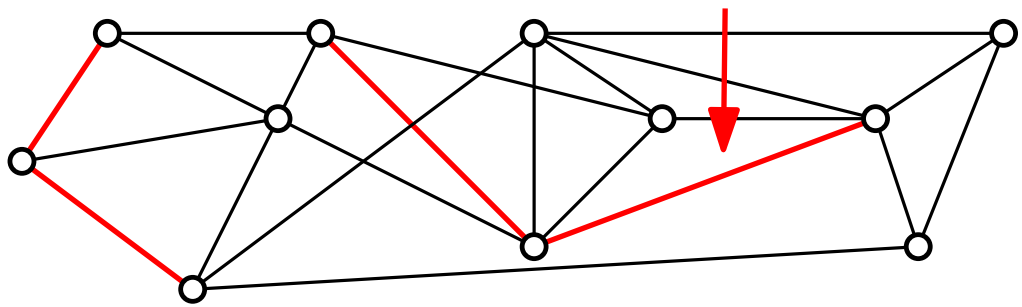

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Answer *basic queries* on d-hop neighborhoods (or d-radius balls)

$$B_d(u) = \{v \mid d(u, v) \leq d\}$$

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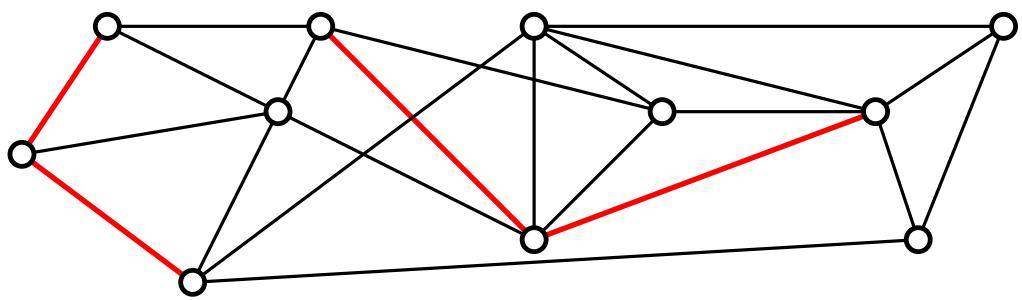

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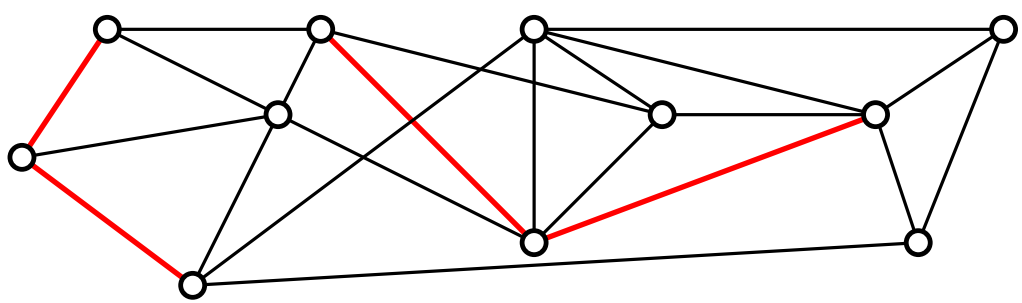

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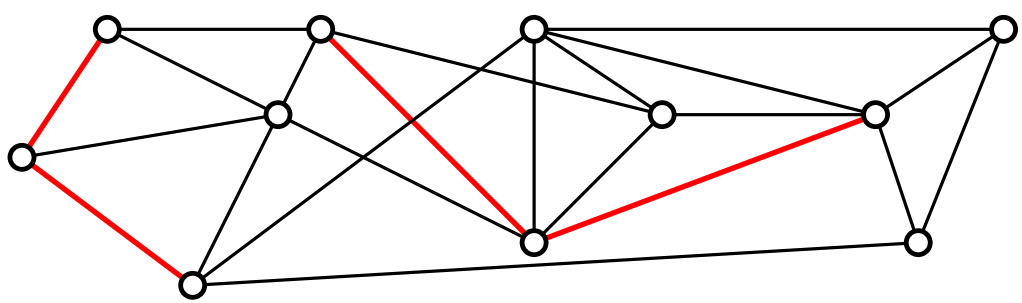

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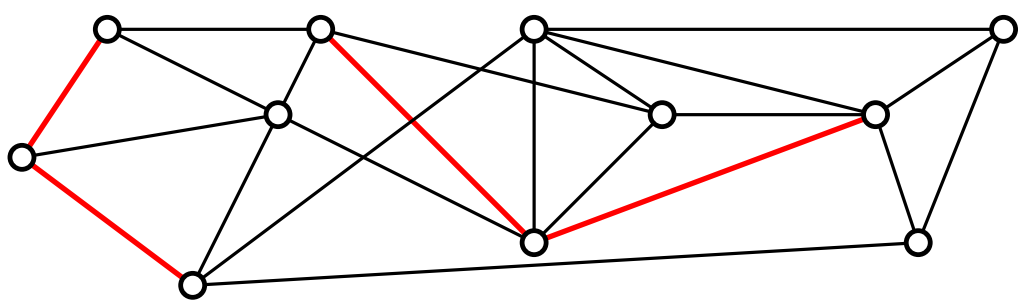

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Layer 2 Downstream tasks

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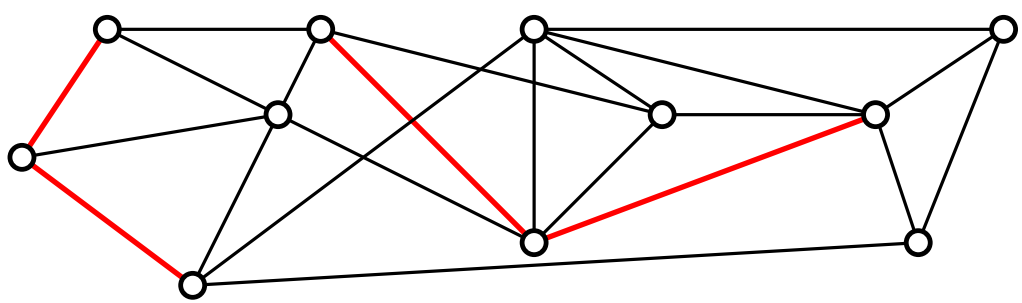

Example of downstream tasks for 2-hop neighborhoods

Approximate top-k central nodes in a (dynamic) network, according to some distance-based indices, e.g.,

Harmonic Centrality

$$HC(v) = \sum_{u \neq v} \frac{1}{d(u,v)}$$

Closeness Centrality

$$CC(v) = \frac{1}{\sum_u d(u,v)}$$

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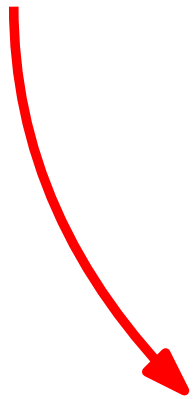

problems overcome with the use of **data sketches**

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iii) (optional) can be **composable**, i.e., given only $\mathcal{S}(A), \mathcal{S}(B)$ we can compute $\mathcal{S}(A \cup B)$

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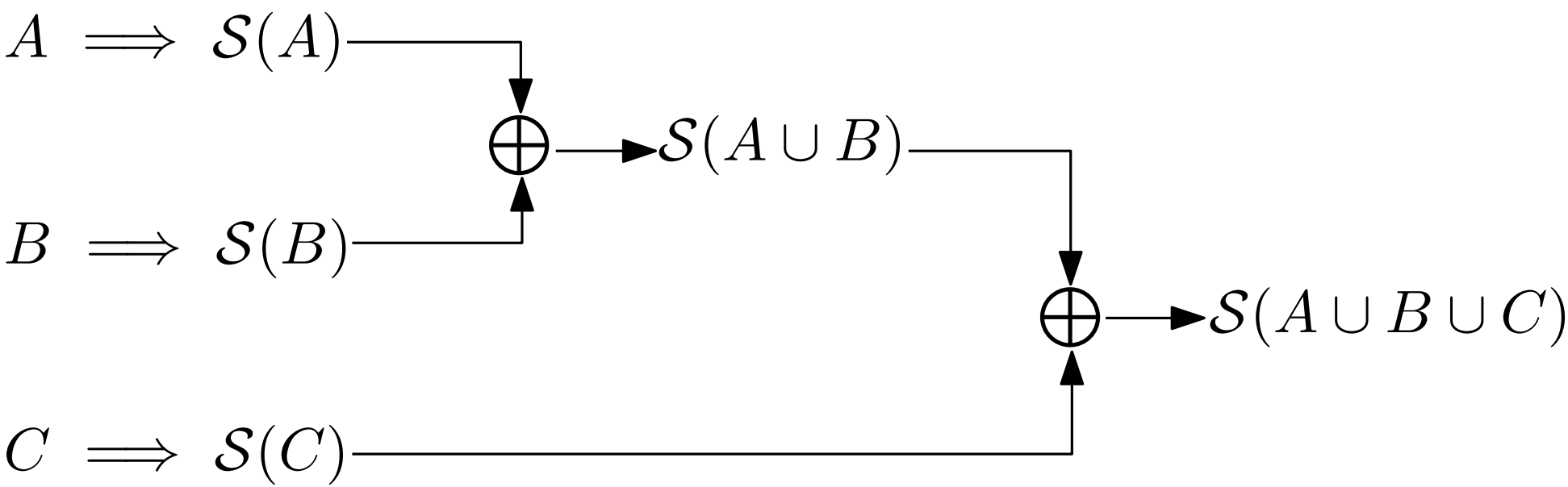

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$\Longrightarrow$ independent of the basic query, and therefore of the downstream task.

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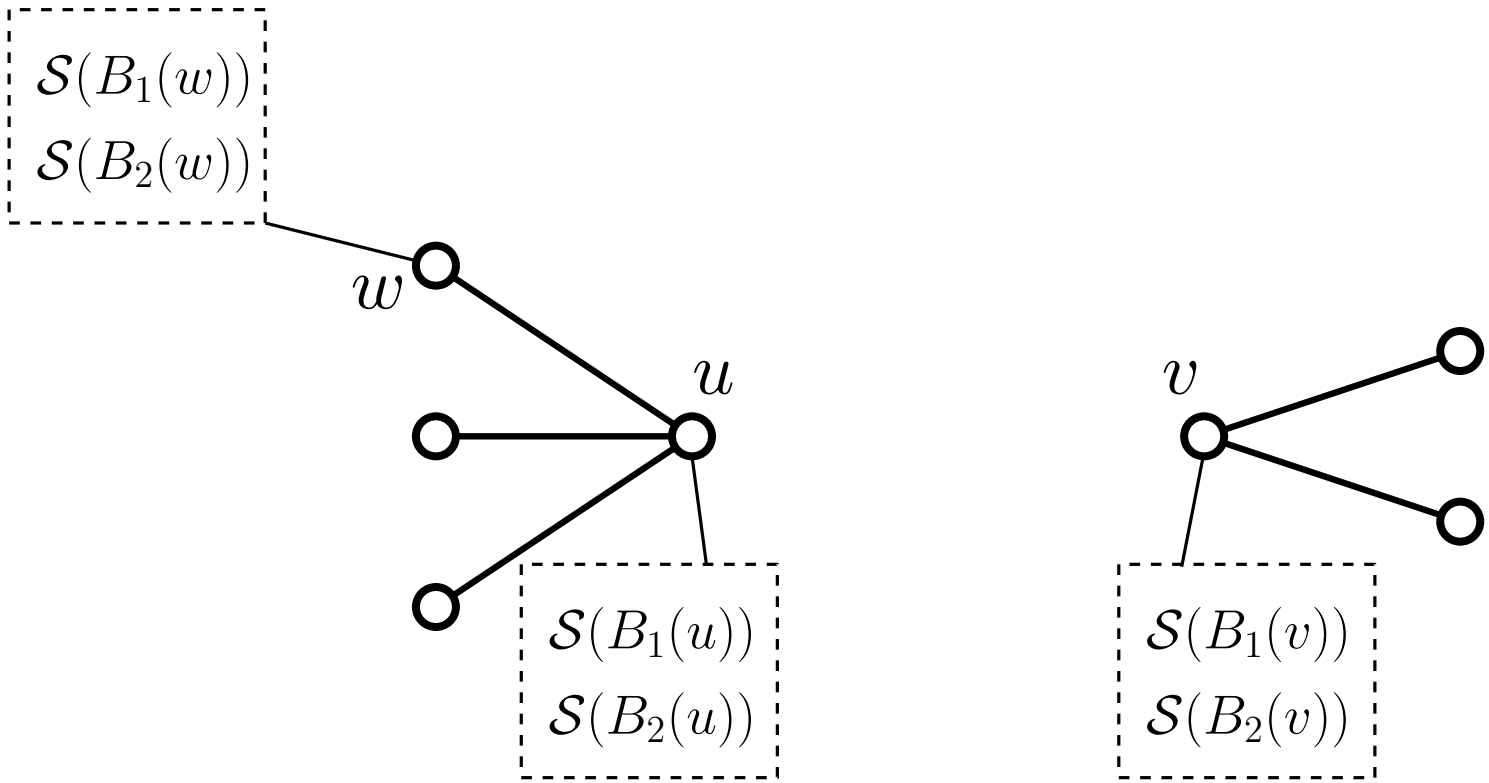

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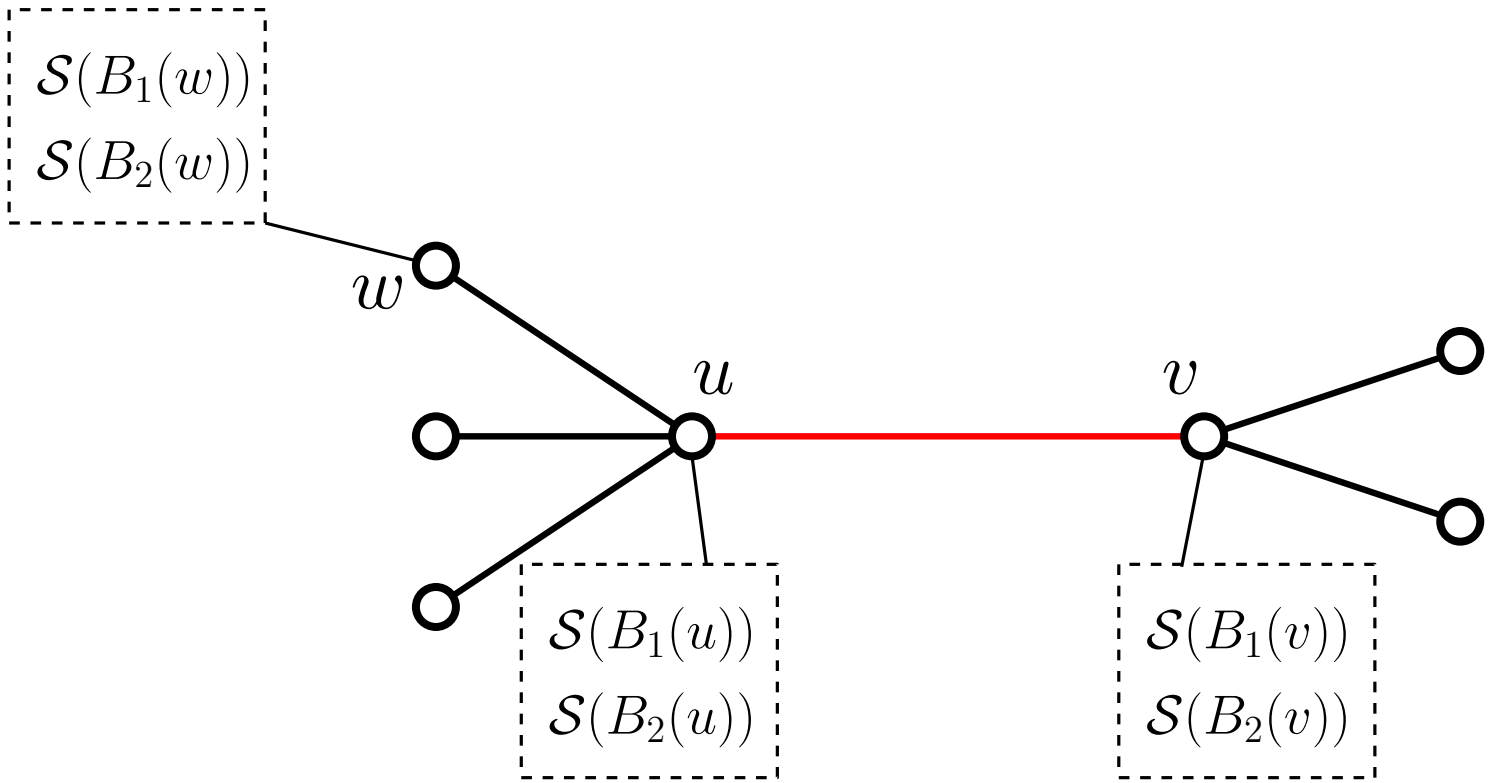

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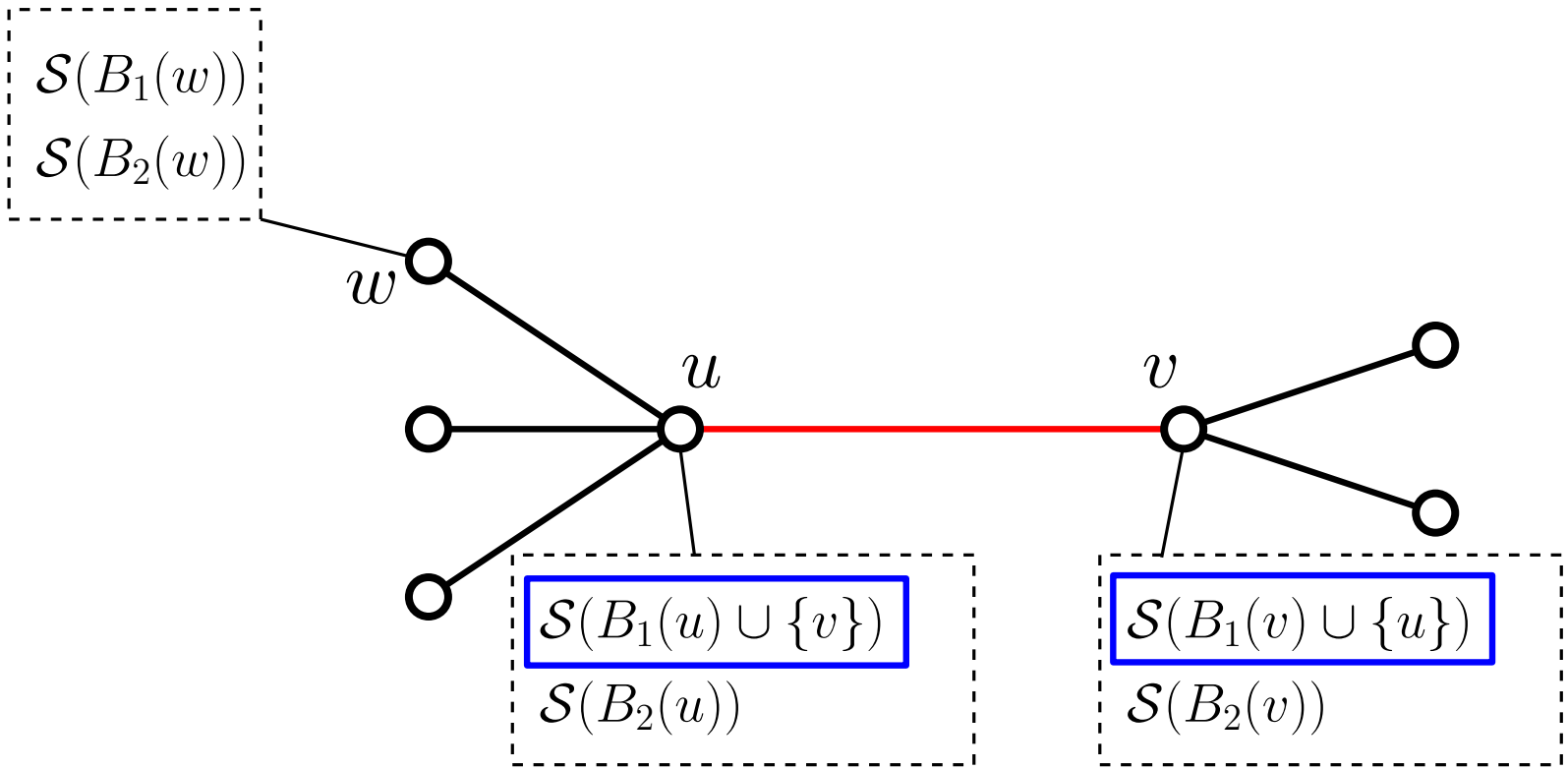

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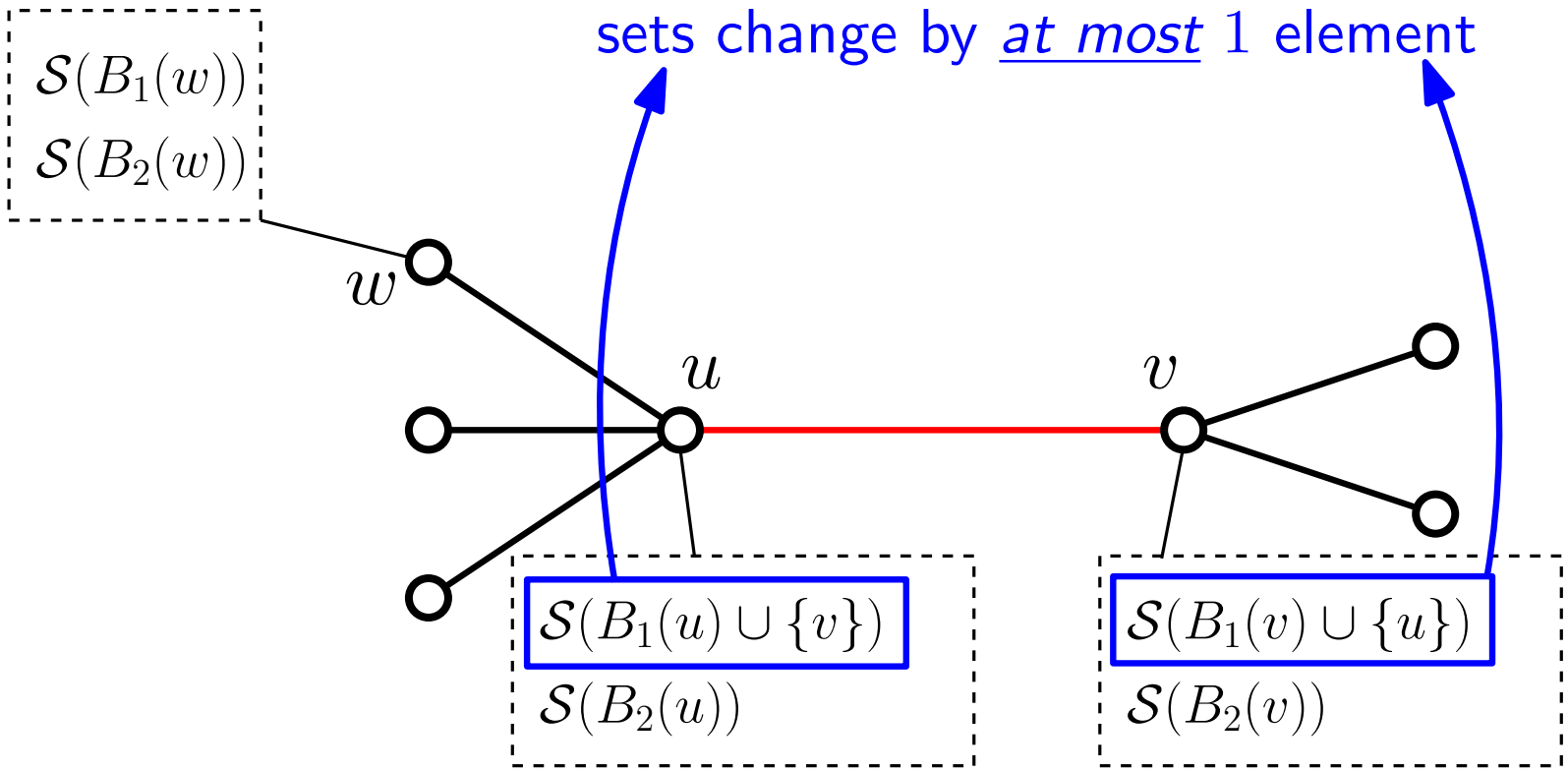

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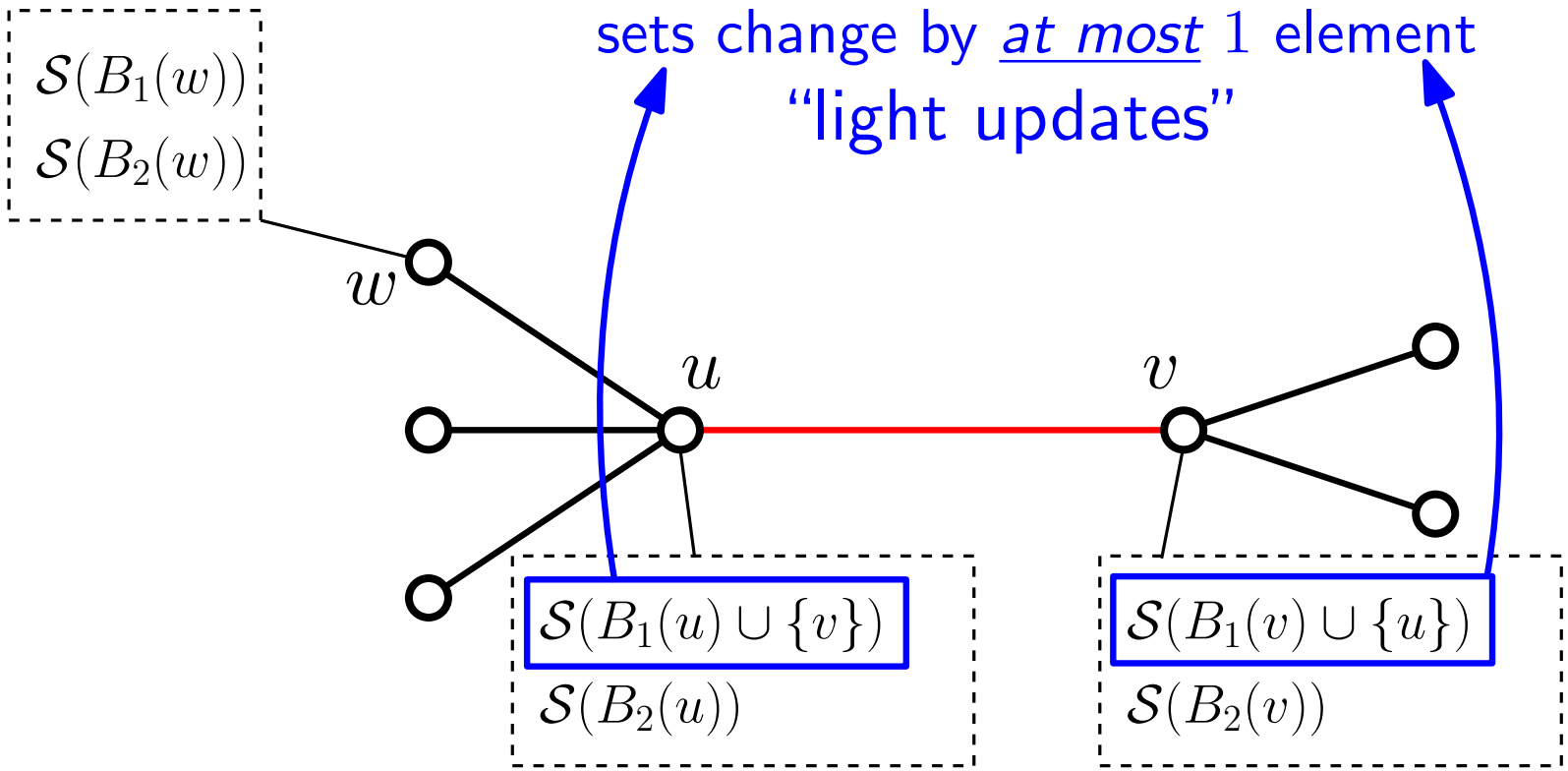

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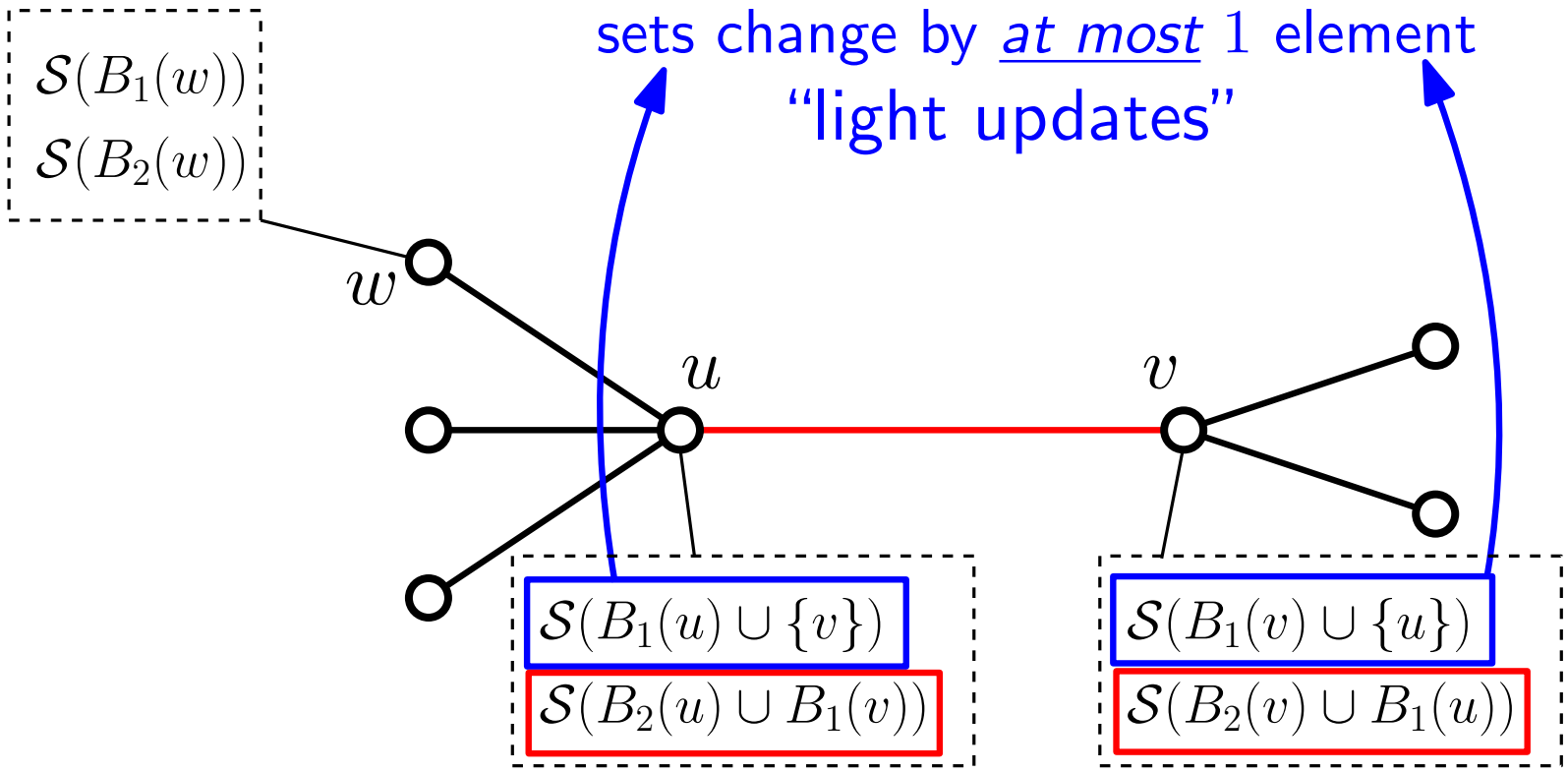

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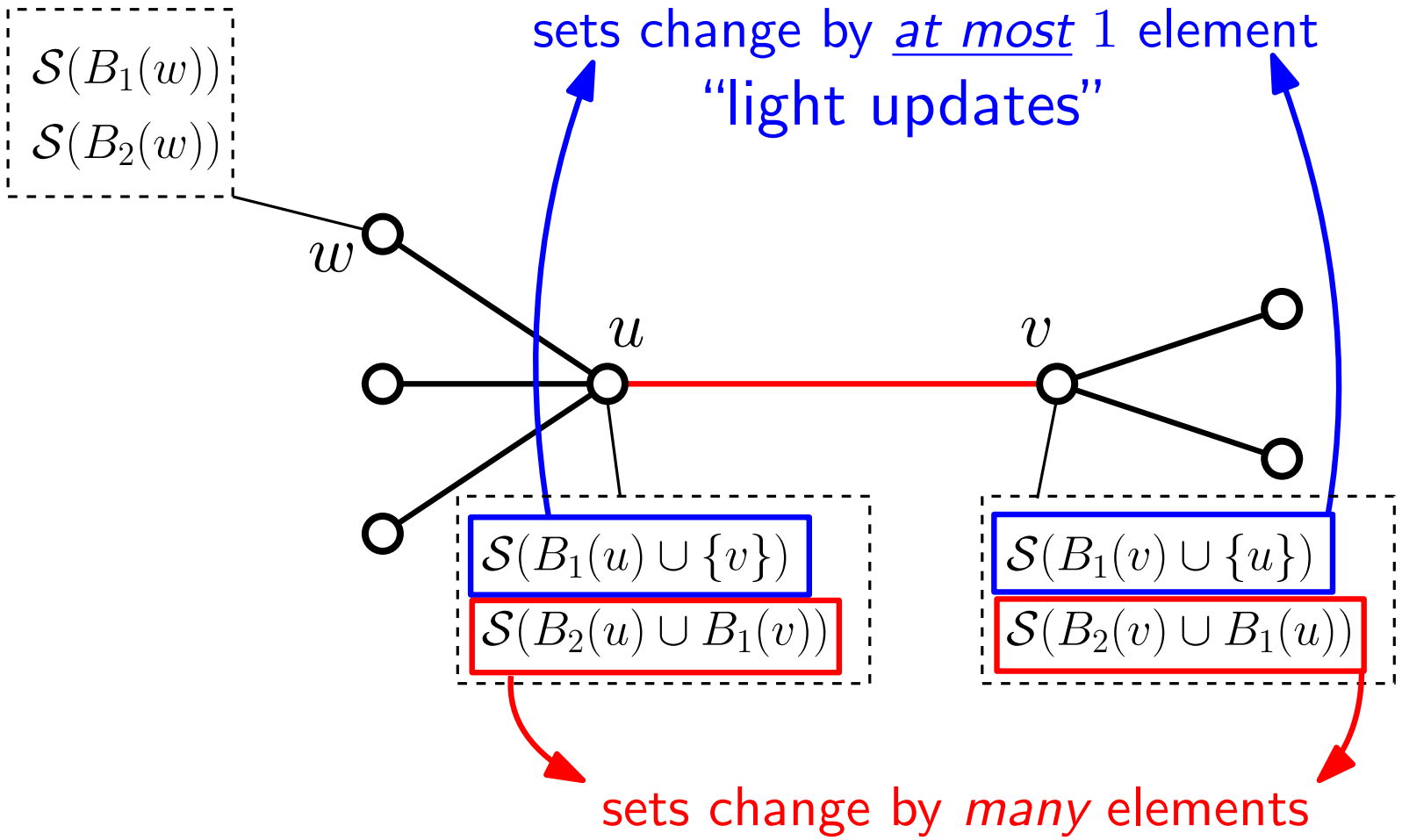

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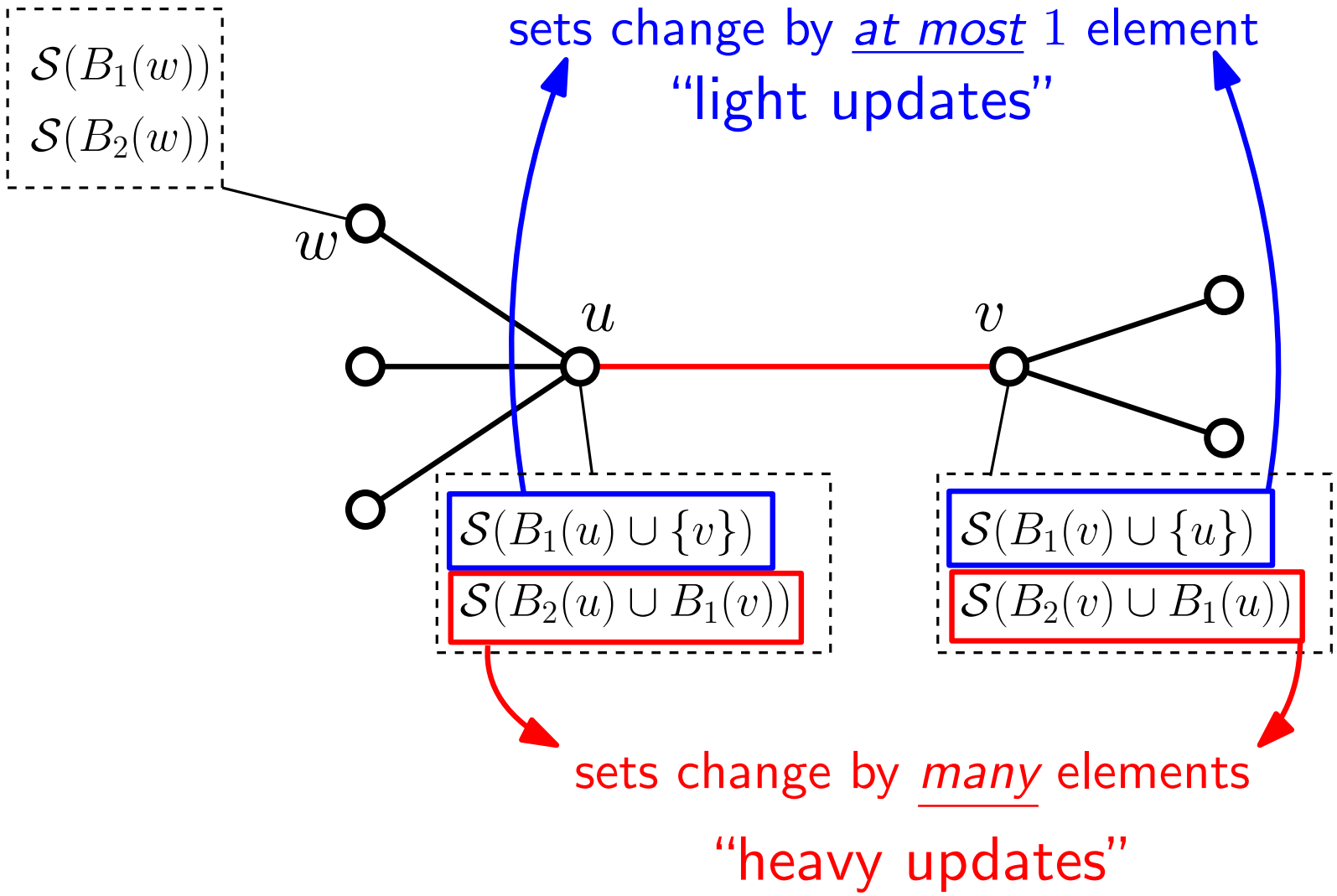

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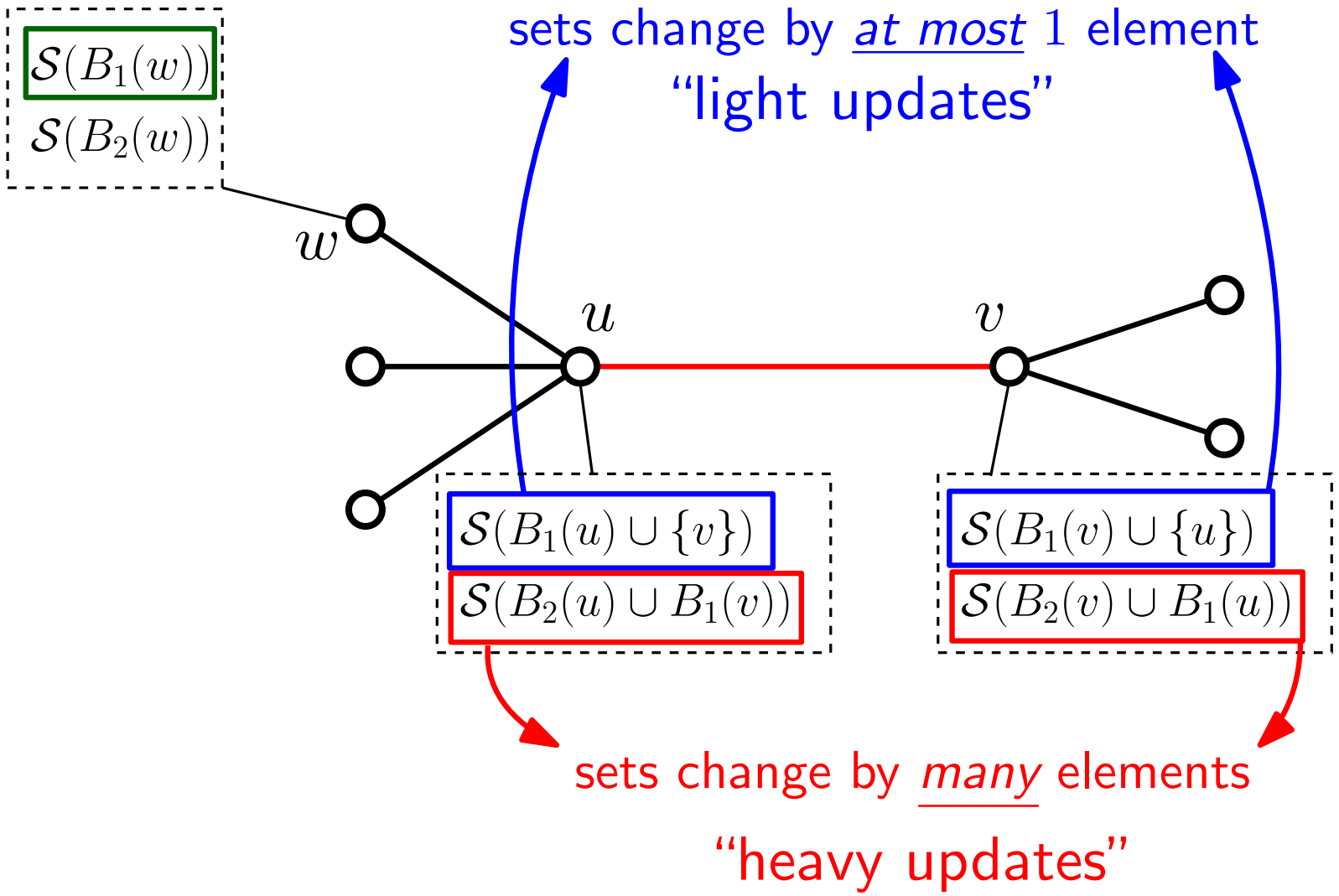

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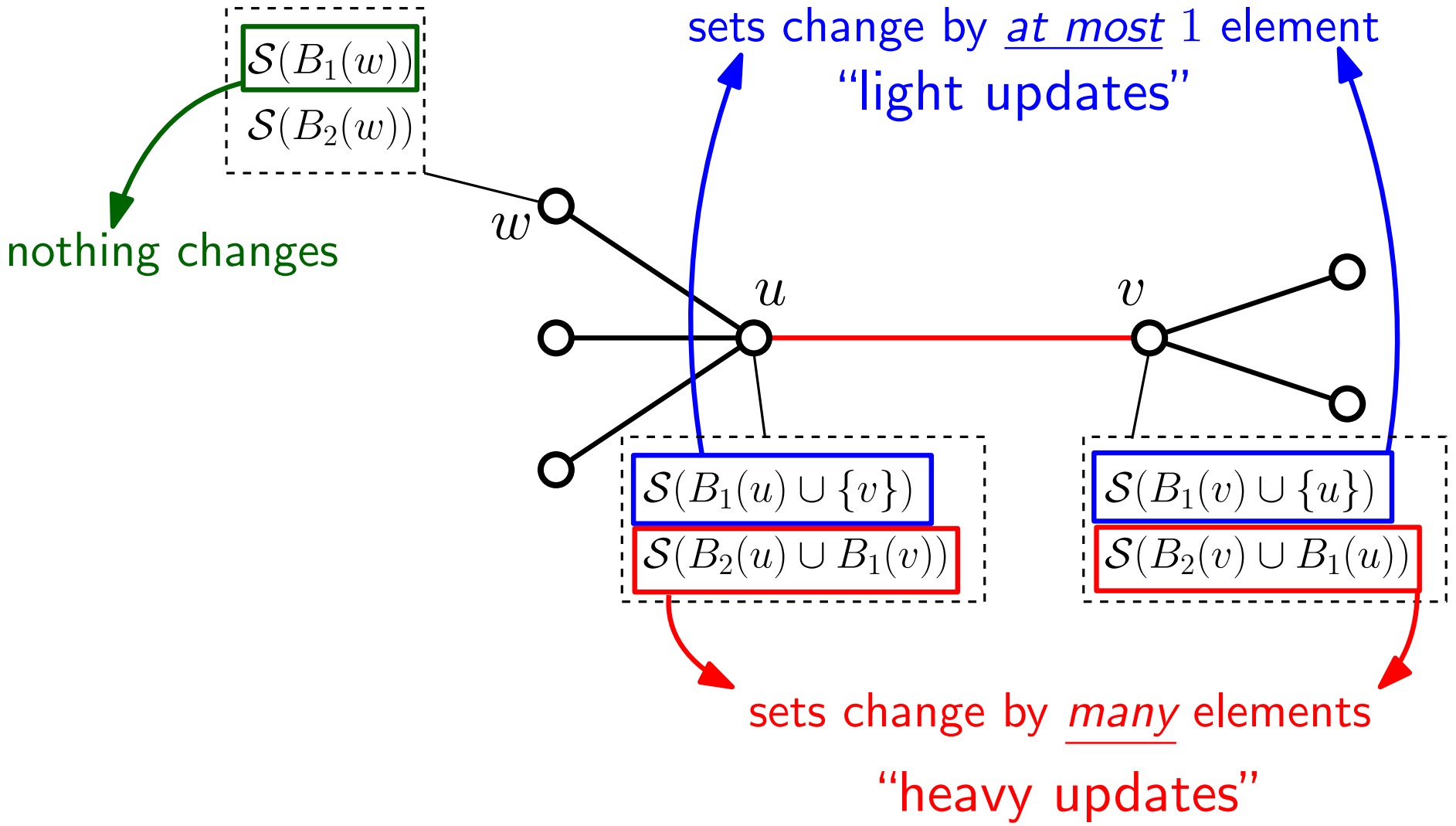

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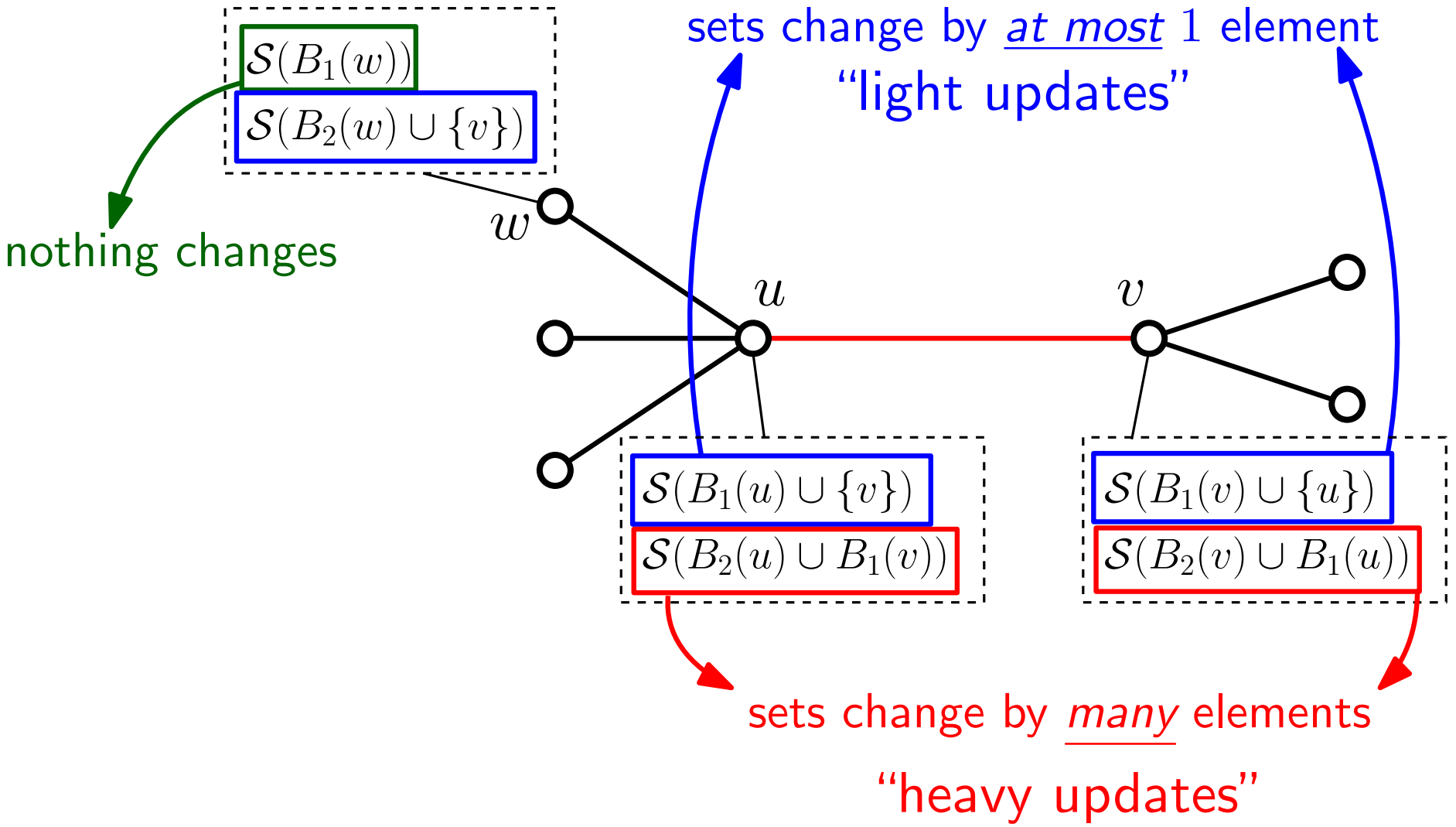

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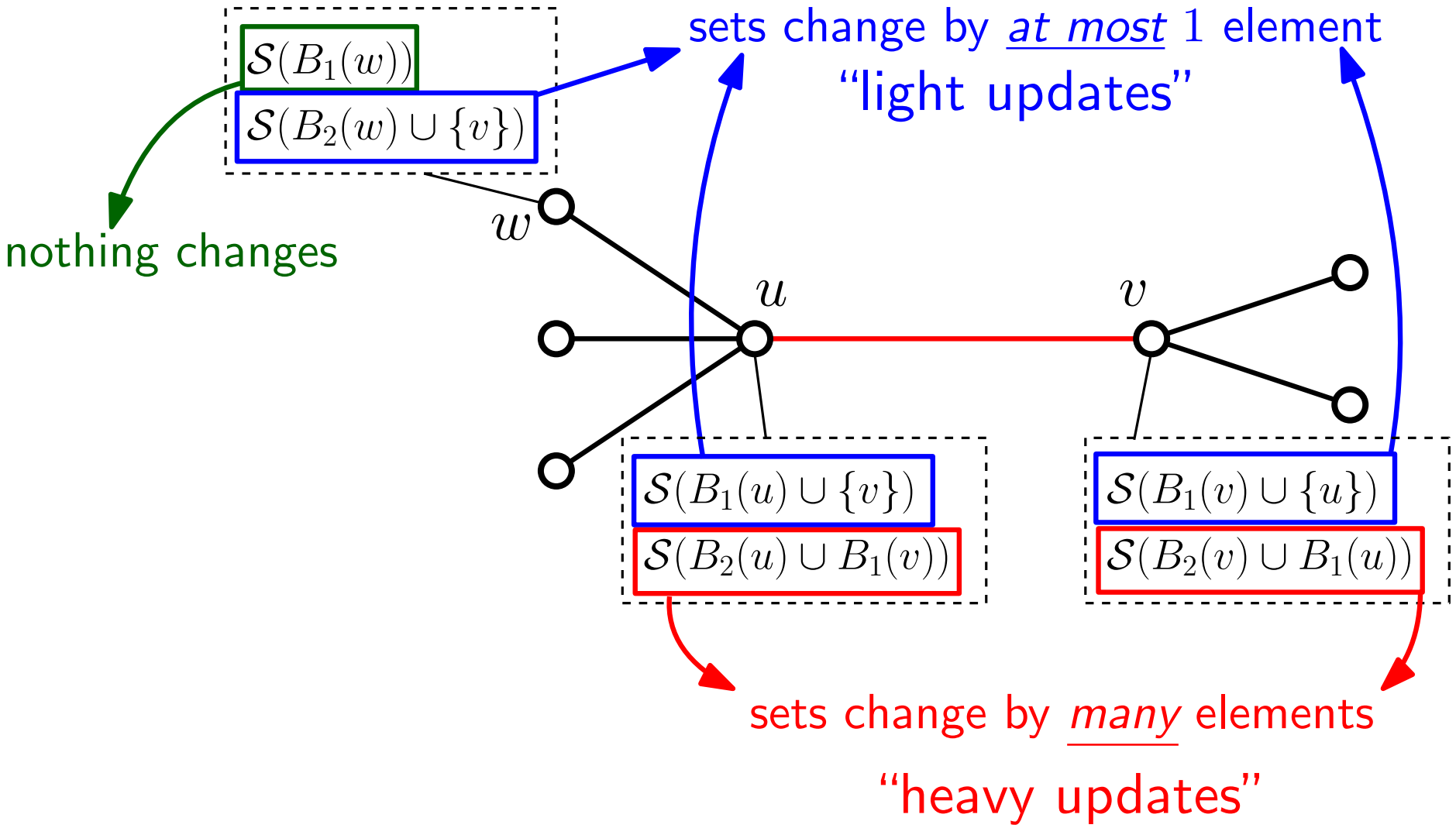

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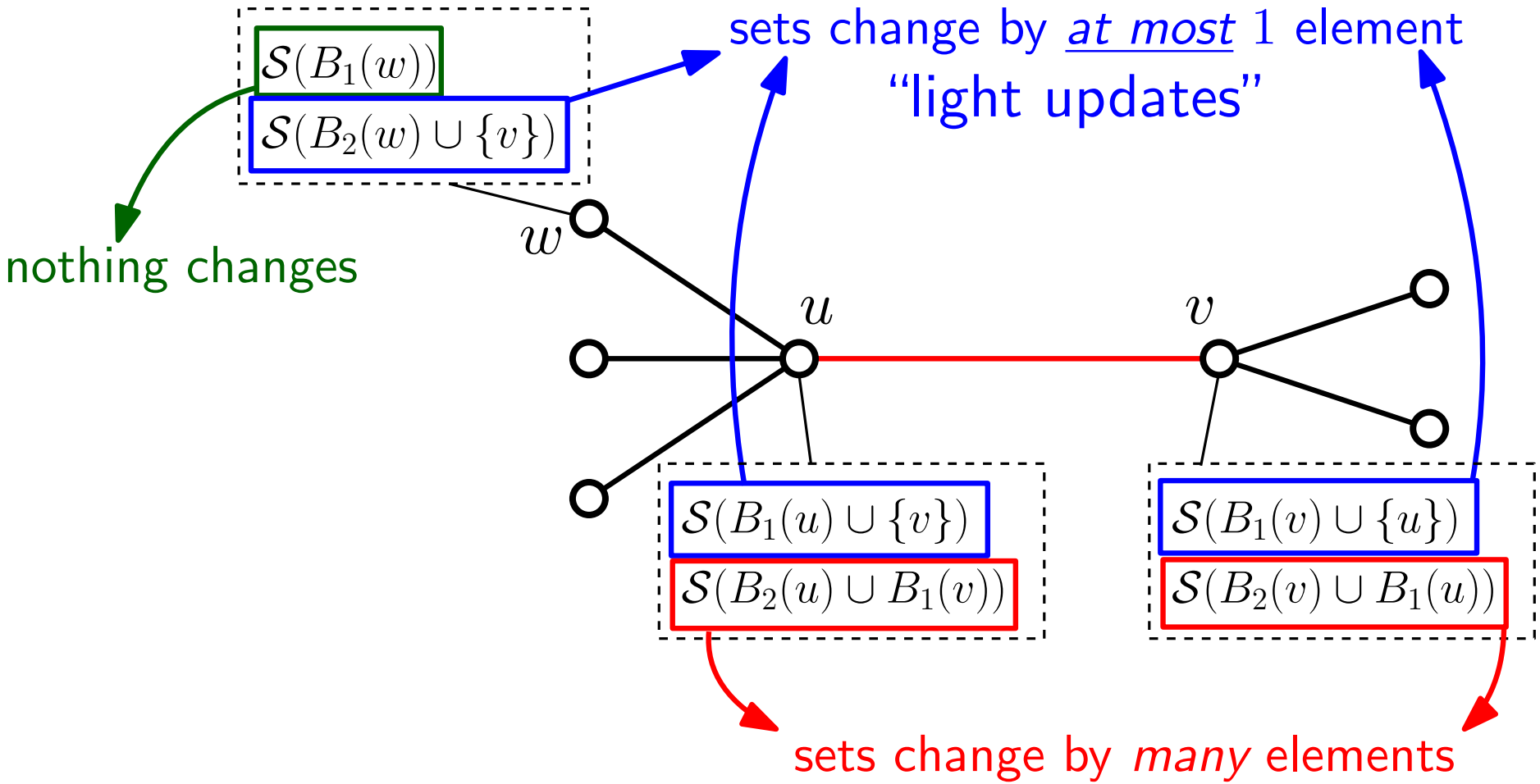

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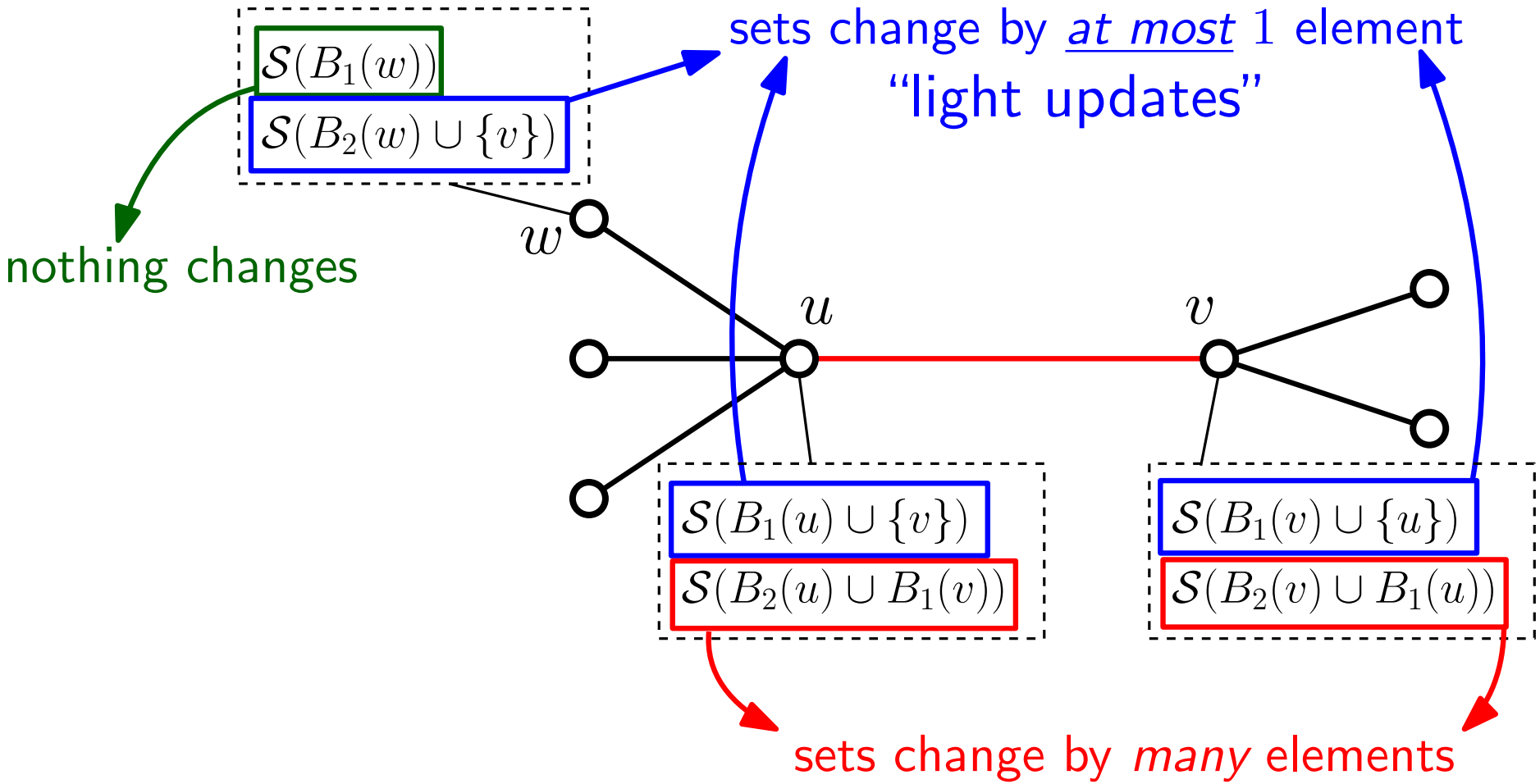

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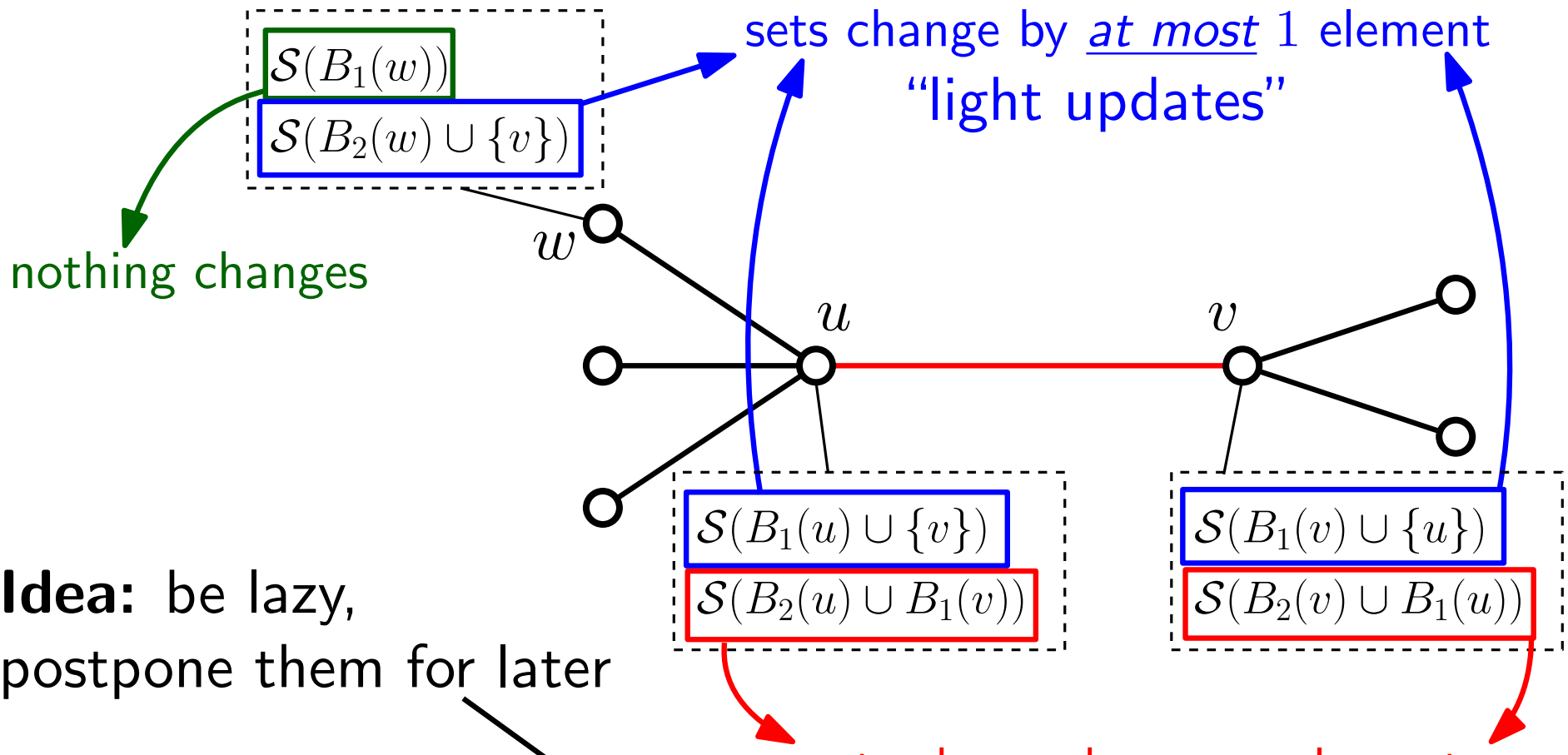

Idea: be lazy,
postpone them for later

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Lazy-Updates Algorithm

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1  Function Insert((u, v)):
2      for x ∈ {u, v} do
3          let y ∈ {u, v} \ {x}
4          B̂1(x) ← B̂1(x) ∪ {y}
           // heavy update
5          B̂2(x) ← B̂2(x) ∪ B̂1(y)
6          δx ← δx + 1
7          if δx ≥ φ · Δx then
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10             foreach z ∈ N(x) do
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14             select k vertices w1, ..., wk ∈ N(x) u.a.r.
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There parameters $\varphi \in (0, 1)$, $k \in \mathbb{N}$ determine the trade-offs between precision and *"laziness"*.

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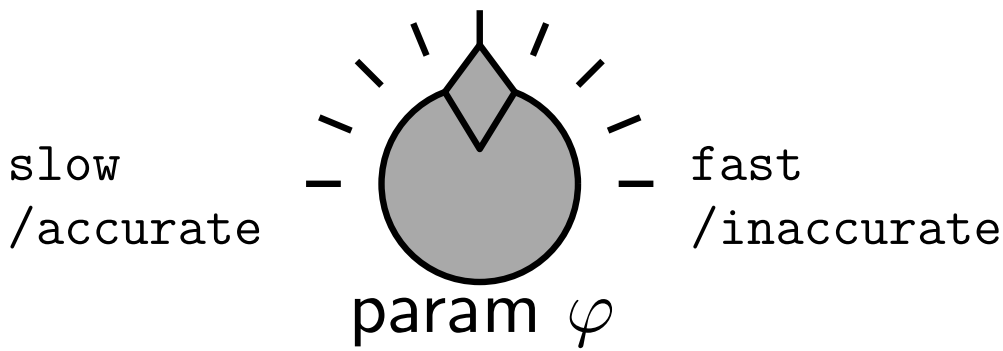

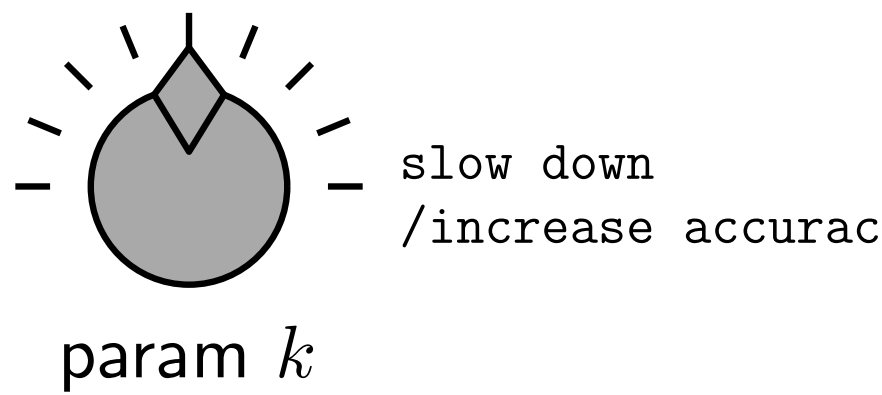

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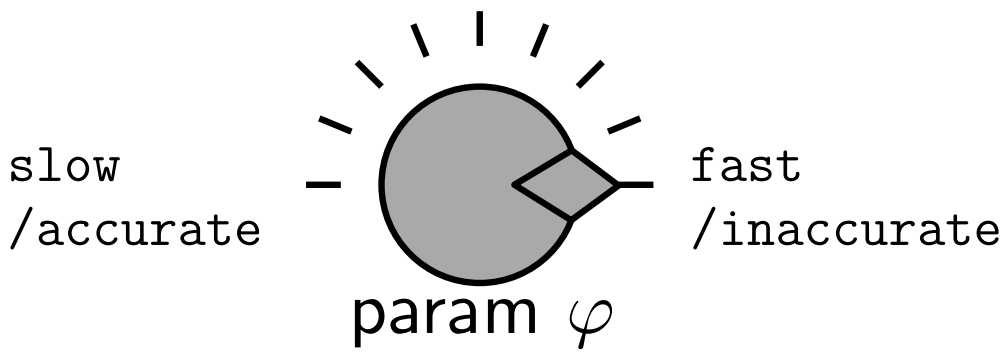

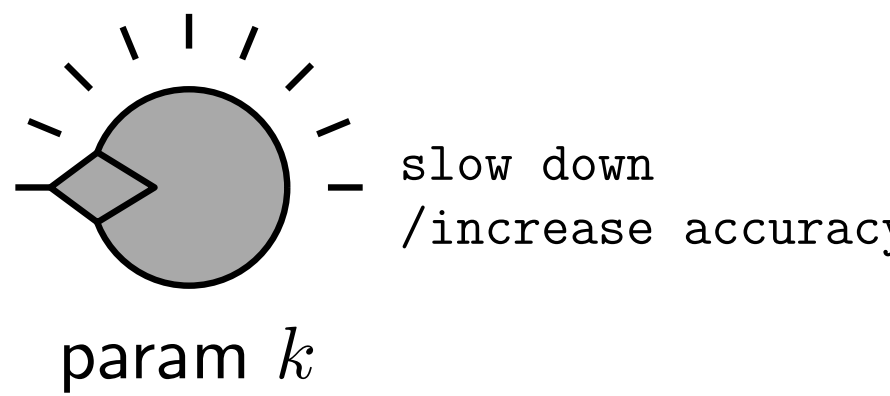

Parameter φ, k are like knobs:

performance regime: $\uparrow \varphi - \downarrow k$

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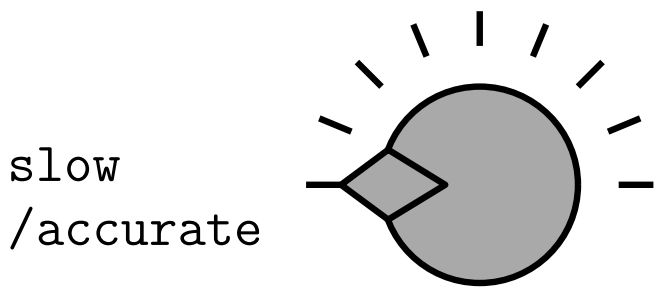

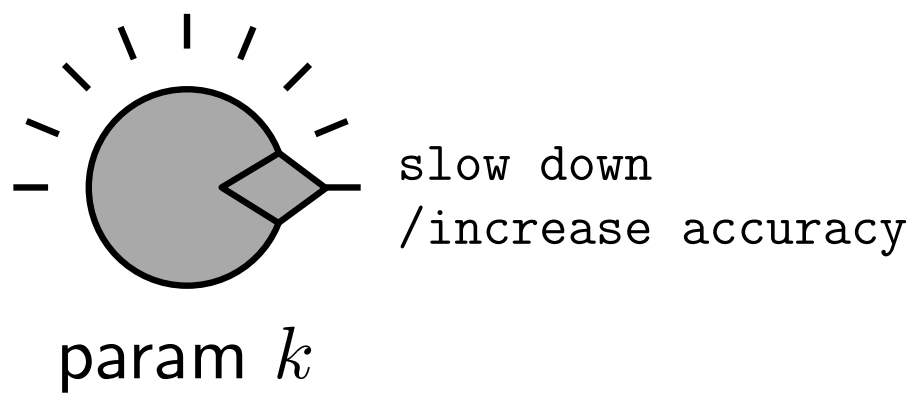

Parameter φ, k are like knobs:

performance regime: $\uparrow \varphi - \downarrow k$

accurate regime: $\downarrow \varphi - \uparrow k$

Main Results

Performance: $O(\frac{1}{\varphi} + k)$ amortized number of sketch compositions.

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Accuracy (under adversarial insertions): Lowerbound of $\Omega(\Delta \cdot (1-\varepsilon)^3)$ sketch compositions if we want $(1-\varepsilon)$-fraction.

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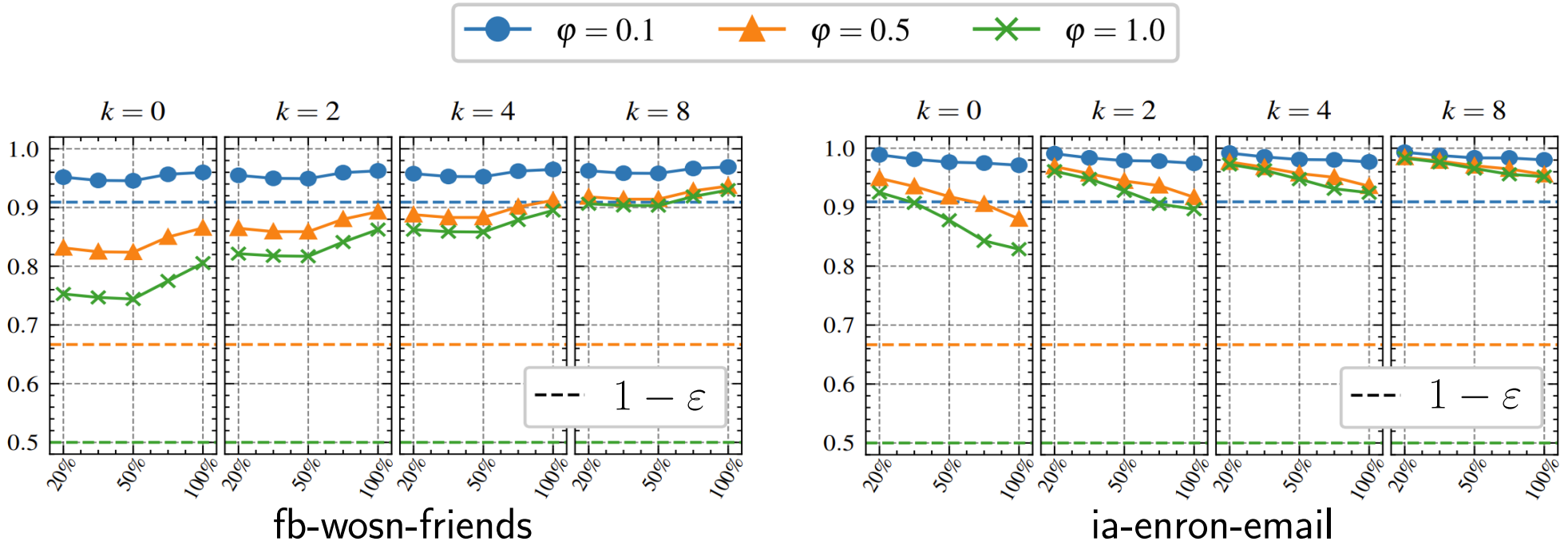
Accuracy (under adversarial insertions): We characterize a broad family of (dense) graphs for which, by choosing the parameters φ, k appropriately, the Lazy-Update algorithm always guarantees a $(1-\varepsilon)$-fraction with high probability, even under adversarial edge insertion streams.

Lazy-Update in practice (real dynamic streams)



fb-wosn-friends

ia-enron-email

linux-kernel-reply

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Basic Query: size estimation of B_2

Sketch: KMV probabilistic counter

Baseline: Keep everything always updated

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	k	φ 0.1	0.25	0.5	0.75	1
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	2	12.00x	18.22x	22.03x	21.49x	23.18x
	4	11.52x	15.34x	17.63x	16.93x	17.54x
	8	9.57x	11.86x	12.80x	12.03x	12.22x
fb-wosn	0	5.48x	9.40x	11.15x	15.23x	16.22x
	2	5.22x	7.35x	7.16x	9.21x	9.67x
	4	4.32x	6.14x	6.05x	7.18x	7.23x
	8	3.79x	4.38x	4.91x	5.31x	5.22x
enron	0	4.17x	5.38x	6.64x	7.12x	7.70x
	2	3.61x	4.11x	4.68x	4.82x	5.27x
	4	3.05x	3.42x	3.78x	3.96x	4.21x
	8	2.51x	2.73x	2.85x	3.01x	3.12x
flickr	0	13.52x	19.97x	26.54x	31.51x	34.20x
	2	12.16x	16.41x	19.52x	21.93x	22.61x
	4	10.76x	13.96x	16.35x	17.17x	17.79x
	8	9.10x	11.19x	12.36x	12.99x	13.28x
youtube	0	41.61x	61.80x	77.08x	86.50x	93.73x
	2	38.16x	51.64x	60.39x	64.93x	65.51x
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	8	33.33x	39.60x	42.73x	43.61x	43.82x

Speed-up wrt baseline

	k	$\varphi = 0.1$	$\varphi = 0.5$	$\varphi = 1$	baseline
linux	0	0.14 ± 0.12	0.19 ± 0.14	0.17 ± 0.11	0.12 ± 0.10
	2	0.13 ± 0.11	0.14 ± 0.10	0.16 ± 0.11	
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	8	0.13 ± 0.11	0.14 ± 0.10	0.16 ± 0.13	
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	2	0.16 ± 0.12	0.12 ± 0.09	0.14 ± 0.10	
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	2	0.16 ± 0.13	0.14 ± 0.10	0.19 ± 0.11	
	4	0.13 ± 0.11	0.13 ± 0.10	0.15 ± 0.11	
	8	0.13 ± 0.10	0.12 ± 0.09	0.15 ± 0.11	

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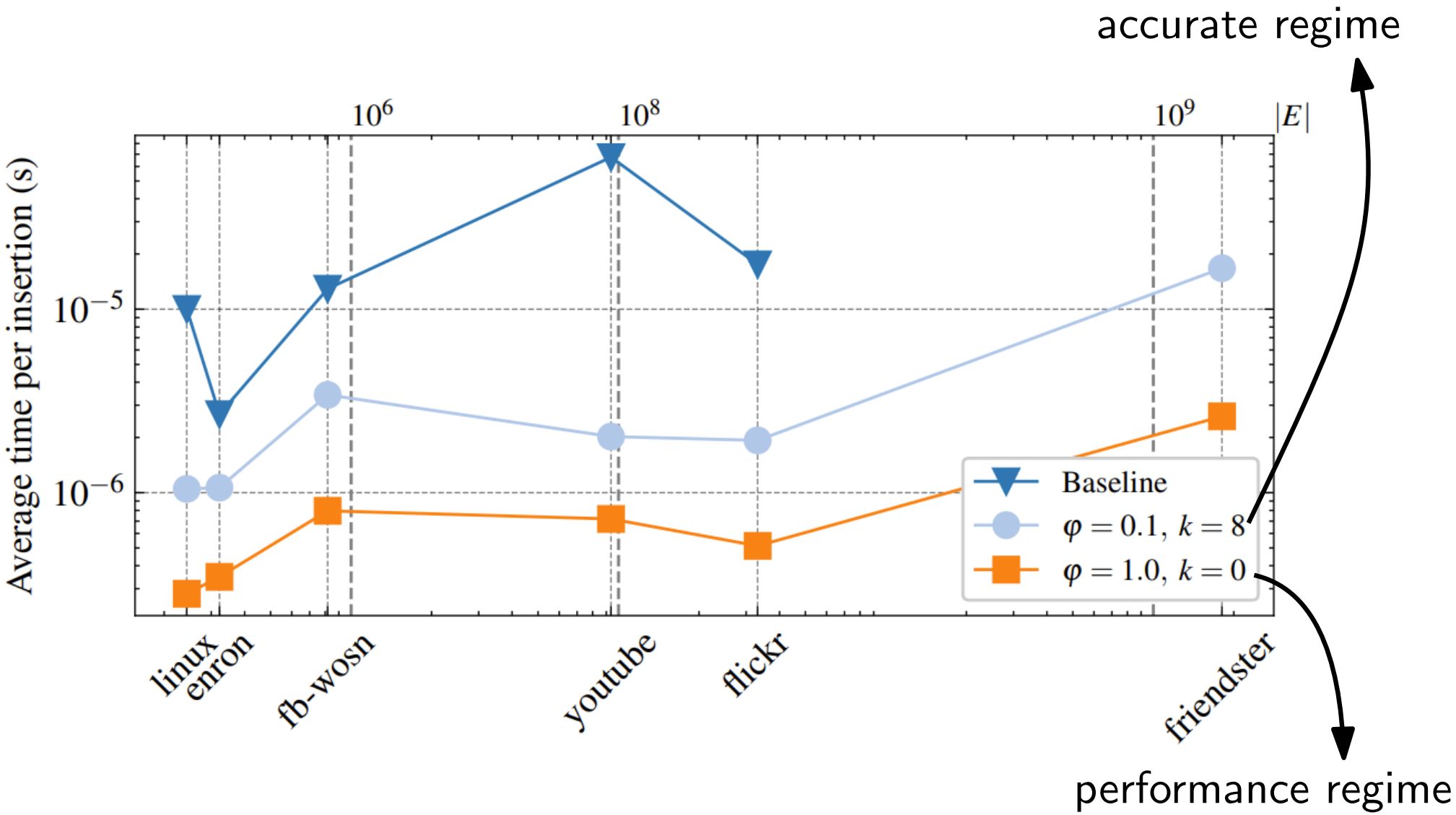
Average time per insertion



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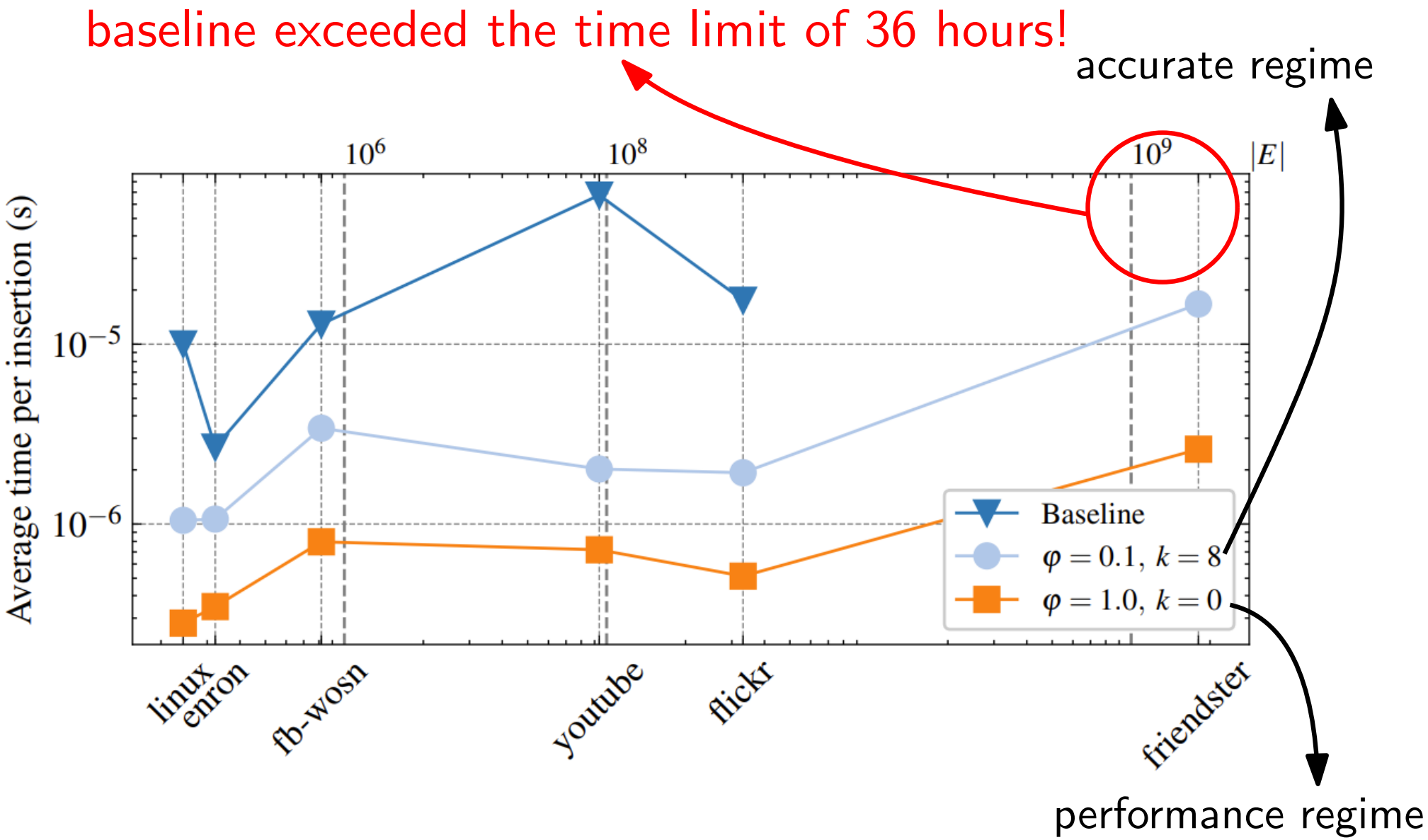
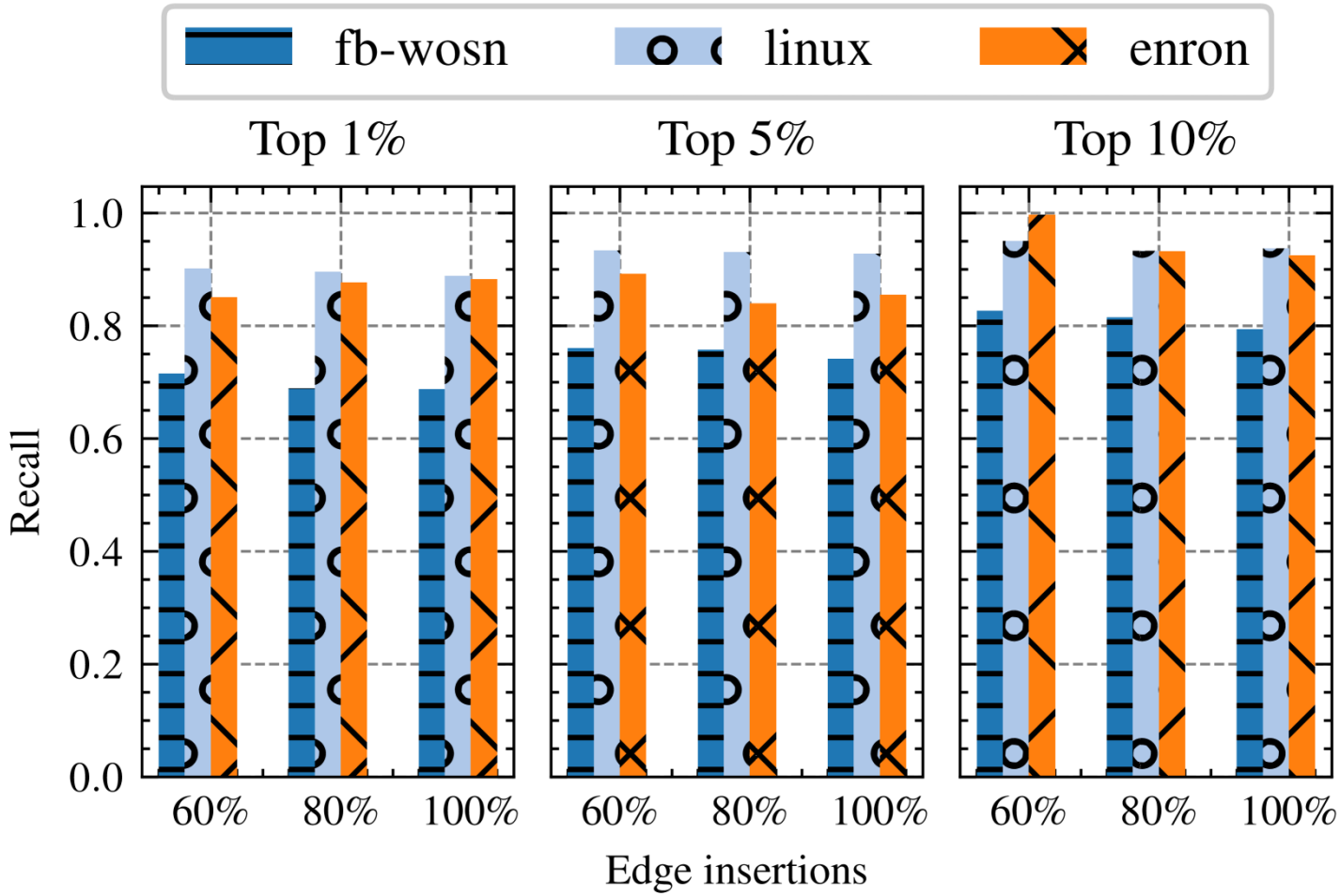


Effectiveness of approximation

Downstream task: maintaining top-k nodes with higher *harmonic centrality* (in a dynamic network).



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ii) Study the case of d-hop neighborhoods with $d > 2$ (even only incremental).

iii) Use our framework to solve *other downstream tasks* beyond those discussed.

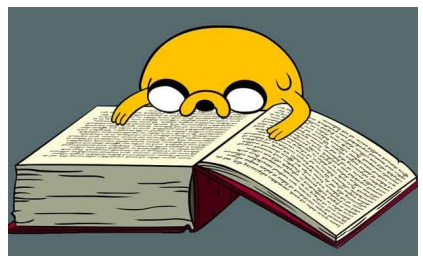
Thanks for your attention

Full version on arXiv

Poster

Presentation

About me



$$L_i(u) = \{v \mid d(u,v) = i\}$$

$$B_i(u) = \{v \mid d(u,v) \leq i\}$$

$$|L_i(u)| = |B_i(u)| - |B_{i-1}(u)|$$

Harmonic Centrality

$$HC(u) = \sum_{v \neq u} \frac{1}{d(u,v)} = \frac{|L_1(u)|}{1} + \frac{|L_2(u)|}{2} + \frac{|L_3(u)|}{3} + \frac{|L_4(u)|}{4} \ldots$$

Truncated Harmonic Centrality

$$HC_2(u) = \frac{|L_1(u)|}{1} + \frac{|L_2(u)|}{2} = |B_1(u)| - 1 + \frac{|B_2(u)| - |B_1(u)|}{2}$$